Quantum Mechanics Students’ Understanding of Mathematical Norms and Normalization

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Introduction

Normalization of vectors, wave functions, and vectors that mathematically represent quantum states is particularly important due to the probabilistic nature of quantum mechanics. Despite this importance, students’ understanding of norms and normalization has not been explicitly studied. In this paper, I present a preliminary framework for students’ understanding of mathematical norms and normalization, with two goals in mind. First, I want to describe students’ understanding of norms and normalization. This conceptual analysis involved an iterative process of developing the framework.

Data Analysis of the nine students consisted of:

- Hour-long, video-recorded, semi-structured interviews with multiple students at three quantum mechanics courses
- Two junior-level linear algebra students and two sophomore-level multivariable calculus students
- Nine physics students from a university in the United States interviewed at the beginning of a senior-level quantum mechanics course
- Two junior-level algebra students and two sophomore-level multivariable calculus students interviewed two-thirds into the semester from a university in the southeastern United States
- While the interviews from all three sites informed the development of the framework, this paper focuses on the nine physics students from the first collection site.
- Data Analysis of the nine students consisted of:
  - Watching the section of the interview where students worked to normalize [3] and [4 − i] writing a summary of each student’s thoughts
  - Coding the transcript of this section for each student
  - Examining how the framework could model each student’s thinking about norms and normalization, modifying the framework as necessary.

Research Questions

1. What understandings of normalization do students have at the beginning of a quantum mechanics course?
2. What aspects of students’ understanding of normalization are particularly useful for making sense of normalization of vectors from unfamiliar vector spaces, such as those encountered in quantum mechanics?

Development of the Framework

- A conceptual analysis (von Glasersfeld, 1995) or “a detailed description of what is involved in knowing a particular (mathematical) concept” (Lockwood, 2013, p. 252) was used in developing the framework.
- This conceptual analysis involved an iterative process of moving back and forth between my own theoretical thinking about the constructs involved in understanding norm and normalization, and student interview data.
- This interview data consists of hour-long, video-recorded, semi-structured interviews with multiple students at three different collection sites:
  - Nine physics students from a university in the northeastern United States interviewed at the beginning of a junior-level quantum mechanics course
  - Nine physics students from a university in the northeastern United States interviewed at the beginning of a senior-level quantum mechanics course
  - Two junior-level algebra students and two sophomore-level multivariable calculus students interviewed two-thirds into the semester from a university in the southeastern United States
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The Framework

- Lack of directionality in the arrow framework is deliberate, as any component could influence how a student thinks about any of the other components.
- Understanding normalization essentially involves three major components, namely: the norm of a vector, procedures for normalizing a vector, and what a normalized vector is.
- Common for students to say the reason for normalizing is so you only have direction and not magnitude.

Model of Danielle’s Understanding of Normalizing

- Students’ understandings of norms and normalization do not necessarily include all of these components and connections; as such, when using the framework to model a student’s understanding, components and connections that are presented in the framework could be more scarce or even missing for a particular student’s model (for instance, see the model of Danielle’s understanding to the right).
- Students’ understanding of norms, particularly the existence of norms other than the Euclidean Norm for real vectors, seems to be especially important for normalizing unfamiliar vectors.

Model of Drake’s Understanding of Normalizing

- Understood normalization as finding the norm, length, or magnitude of a vector.
- Understood multiple representations for real vectors.
- Understood several equivalent ways to find the Euclidean Norm of a real vector.
- Seemed to only know of one norm, namely the Euclidean Norm for real vectors.
- When asked to normalize the unfamiliar vector [4 − i], Danielle tried taking the square root of the dot product of the vector with itself. When this resulted in the square root of a complex number, Danielle was unsure of her answer, but explained that no intuition was readily available, particularly because she had no way of graphically visualizing this vector.
- Did have knowledge of unit vectors, but normalizing did not seem to be connected to the idea of creating or finding these unit vectors.

Conclusions & Future Directions for Research

We can often think about normalizing vectors as simple, something that should not create much trouble for students. However, students’ understanding of normalization can be quite intricate, and the unfamiliar vector spaces in quantum mechanics, such as C^n to model Spins, can present unique challenges to students. In future research, I hope to explore students’ understandings of normalizing wave functions, as well as examine ways in which instructors might help their students better understand normalization and its importance within quantum mechanics and other STEM applications.

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